

Productivity Dispersion and Monetary Policy

Erfan Ghofrani ^{*†}

First Draft: June 2022

Abstract

I present a theoretical framework that features contractionary productivity dispersion shock which is a result of the interaction between substitutability of supplied labor and demanded goods. I introduce information friction as a source of nominal rigidity to study the impact of the productivity dispersion shock on the conduct of monetary policy. In particular, I assume firms have incomplete information about the productivity dispersion when they set the price. I show that in the environment with nominal rigidity, replicating full-information flexible price equilibrium is always feasible and optimal, however, the optimal policy is not an inflation targeting policy. The optimal monetary policy is the policy which eliminates the dependence of the idiosyncratic nominal variables on the unknown productivity dispersion and as a result makes the information friction irrelevant.

Keywords: Uncertainty, Volatility, Dispersion, Optimal monetary policy, Information friction, Expectations.

JEL Codes: D83, D84, E32, E51, E52.

^{*}Department of Economics and Business, Pompeu Fabra University.

[†]I am grateful to my advisors Vladimir Asriyan and Alberto Martin for for their invaluable support. I express my gratitude to Isaac Baley, Davide Debortoli, Jordi Galí, Edouard Schaal, Victoria Vanasco as well as all the participants at the CREi macro lunch for their comments that substantially improved the paper.

1 Introduction

Originated by Bloom (2009), a growing branch of literature studies the impact of the second moment shocks on business cycles. Berger and Vavra (2017), Jurado et al. (2015), Baker et al. (2016) and Fernández-Villaverde et al. (2011) are among papers that show that second moment shocks are counter-cyclical. Many theoretical frameworks rationalize this evidence through wait-and-see effects¹, risk premium effects² or precautionary motives³.

Dispersion shock is one form of the second moment shocks that plays an important role in business cycles. Kehrig (2015), Bachmann and Bayer (2014), Bachmann and Bayer (2013) and Bloom et al. (2018) provide evidence that dispersion shocks are counter-cyclical. Counter-cyclical productivity dispersion, which is a well established empirical fact, is absent in many friction-less models. According to the well known Oi-Hartman-Abel⁴ effect second moment productivity shocks, due to the complementarity channel between the productivity and factors of production, are expansionary in models without friction. In this paper, in the first step, I build a friction-less model that departs from the Oi-Hartman-Abel effect and features contractionary productivity dispersion shock.

The proposed model is based on a simplified version of the framework that is introduced by Angeletos et al. (2020). In my static model there is a representative household consisting of a consumer and a continuum of workers who supplies labour to a continuum of firms that produce differentiated goods. I obtain the contractionary dispersion shock in the friction-less model by introducing taste for variety and substitutability, not only for the consumed goods, but also for the supplied labor in the aggregate economy. For a small degree of substitutability of either supplied labor or consumption good, the dispersion shock will be contractionary. The main intuition is that for small values of substitutability, the model converges to the Leontief environment. As a result, the standard complementarity channel between factor of production and the productivity is broken and the second moment shock is not expansionary anymore.

In the next step, I study the impact of the dispersion shock on the conduct of the optimal monetary policy by using the information friction as a source of nominal rigidity. Information friction refers to the scenario that at the time that firms set the price, they have incomplete (and *not* asymmetric) information about underlying aggregate productivity dispersion. Chosen prices are fixed and can not be updated after receiving more information. Following Angeletos et al. (2020) I will call the scenario in which the model features information driven nominal rigidity *sticky price* and the environment in which the price is set with complete information *flexible price*. Using information friction as a source of nominal rigidity is not the contribution of the paper, it has been widely used in the literature before⁵. The main contribution of the paper is the introduction of the uncertainty about the dispersion when prices are chosen.

In addition to the information friction, the model has another source of the distortion and that is the monopolistic competition. Please note I refer to the environment without information friction as friction-less because the welfare loss due to the monopolistic competition can simply be eliminated by implementing the standard optimal fiscal pol-

¹see Bloom (2009), Bachmann and Bayer (2013) .

²see Arellano et al. (2016) and Christiano et al. (2014).

³see Basu and Bundick (2017), Leduc and Liu (2016) and Ravn and Sterk (2017).

⁴see Oi (1961) , Hartman (1972) and Abel (1983).

⁵see Mankiw and Reis (2002), Mackowiak and Wiederholt (2009), La'O and Tahbaz-Salehi (2022) and Angeletos and La'O (2020)

icy. Both monetary and fiscal policy makers are restricted and committed to follow pre-determined rules which are contingent on the realized states.

It is a well known fact that in the absence of information driven nominal rigidity, monetary policy is neutral. However, in the environment with information friction monetary policy has real effect. I show that the optimal monetary policy is the policy that replicates the full-information flexible price scenario, however, it is not an inflation targeting (aggregate price stabilizing) policy. The policy that replicates the full-information scenario is always feasible and basically eliminates the dependence of idiosyncratic nominal variables on the aggregate dispersion shock and stabilizes idiosyncratic prices, however, this policy does not stabilize the aggregate price level. In other words, this policy is equivalent to the policy in the flexible price environment that makes the idiosyncratic prices, but not the aggregate price, irrelevant of the productivity dispersion. So if in the full information environment idiosyncratic marginal cost and prices do not depend on the dispersion, in the environment with incomplete information, the unknown term does not play any role neither and the information friction is eliminated.

Reducing uncertainty in the market and eliminating information friction as an optimal policy is in contrast with “Paradox of Transparency” literature ⁶. The optimal policy in my paper, that eliminates the information friction, is in line with Kohlhas (2022), however, it is *not* optimal because of the increase in the informativeness of prices nor reduction in the uncertainty of the central bank.

The structure of the paper is as follows. In the section 2, I introduce the baseline model without any nominal rigidity and study the equilibrium. In the section 3, we see how we can obtain the contractionary dispersion shock in the friction-less model. In the section 4, I introduce information friction as a source of nominal rigidity and study the equilibrium. In the section 5, I study the optimal monetary policy in the environment with nominal rigidity and finally I conclude in the section 6.

2 Baseline model without nominal rigidity

In this section I present the baseline full information model in which there is not any source of nominal rigidity. I show how the interaction between substitutability of supplied labour and demanded goods can generate the contractionary productivity dispersion shock. The model is based on Angeletos et al. (2020).

2.1 Environment

The model is a one-period and static.

Household:

There is a representative household consisting of a consumer and a continuum of workers who supplies labour to a continuum of firms, indexed by $i \in I = [0, 1]$. The household maximizes the utility:

$$U = \frac{C^{1-\gamma} - 1}{1 - \gamma} - \frac{N^{1-\epsilon} - 1}{1 - \epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1 - \delta}$$

⁶see Morris and Shin (2005), Amador and Weill (2010), Ou et al. (2021) and Gaballo (2016).

where C is the aggregate consumption basket, N is the aggregate supplied labour and $\frac{M}{P}$ is the real money in the utility. $\gamma > 0$ parameterizes the income elasticity of labor supply and the risk aversion, $\epsilon < 0$ parameterizes the Frisch elasticity of labor supply ⁷ and $\delta > 0$ parameterizes the convexity of the utility with respect to the real balance. Aggregate consumption, labour and price are determined by the following CES aggregators.

$$C = \left[\int_I (c_i)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}, \quad N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di \right]^{\frac{\omega}{\omega-1}}$$

$$P = \left[\int_I (p_i)^{1-\rho} di \right]^{\frac{1}{1-\rho}}, \quad W = (1 - \tau) \left[\int_I (w_i)^{1-\omega} di \right]^{\frac{1}{1-\omega}}$$

where c_i is the consumed quantity of the commodity produced by the representative firm i at the price p_i and $\rho > 1$ is the elasticity of substitution between different consumed goods. n_i is the supplied labor for the production of the good that is produced by the firm i with wage w_i and $\omega < 0$ is the elasticity of substitution between different supplied labors.⁸

The representative household receives labor income and profits from all firms in the economy. Its nominal budget constraint is thus given by:

$$\int_I p_i c_i di + M = \int_I \Pi_i di + (1 - \tau) \int_I w_i n_i di + T$$

where M is nominal demanded money, Π_i is the profit from the firm i .

Government:

There is a government which collects tax and redistribute it in a lump-sum fashion. In the household's budget constraint τ is the constant tax rate on the labour income and T denotes the lump-sum redistribution tax ⁹. The government plays the role of the central bank at the same time and supply the nominal money M ¹⁰. The government's budget constraint is:

$$T = \int_I \tau w_i n_i di + M$$

Firms:

The output of the representative firm in island i is given by:

$$y_i = A_i n_i$$

A_i is the productivity of the firm i and n_i , which is the only factor of production, is demanded labour for the production of the good i . Firms produce differentiated goods in monopolistic competitive fashion. The firm's realized profit is given by:

$$\pi_i = p_i y_i - w_i n_i$$

Markets clearing:

Labour supply in each firm i is equal to the labour demand at the market clearing wage

⁷Please note $\frac{-1}{\epsilon}$ is the Frisch elasticity of labor supply.

⁸I assume the household does not only have a taste for variety for the consumption good but also the supplied labor.

⁹ τ is always chosen optimally such that the monopolistic competition distortion will be eliminated.

¹⁰The monetary policy rule will be discussed in the section of the model with nominal rigidity.

w_i . Demand and supply for the produced goods of each firm i are equal at the market clearing price p_i .

Idiosyncratic productivity shocks:

As it was mentioned earlier A_i is the productivity of the firm i . It is log-normally distributed in the cross-section of firms:

$$a_i = \log(A_i) \sim N\left(\bar{a} - \frac{\sigma^2}{2}, \sigma^2\right)$$

Idiosyncratic log-productivities are centered around $\bar{a} - \frac{\sigma^2}{2}$ and σ^2 captures the degree of dispersion in the productivity between different firms. This form of distribution guarantees that the second moment dispersion shock is only second moment shock and is not affecting average productivity $E(A_i) = \exp(\bar{a})$. $\bar{a}$ is predetermined and known to everyone. $\sigma^2 \sim IG(\alpha_0, \beta_0)$ is drawn from the Inverse-Gamma distribution with the shape parameter $\alpha_0 \rightarrow \infty$ and the scale parameter $\beta_0 \rightarrow \infty$ such that $\frac{\beta_0}{\alpha_0} \rightarrow \sigma_0^2$ ¹¹. In the baseline model without information friction, firms perfectly observe the realizations of σ^2 before price setting¹².

2.2 Equilibrium

The equilibrium consists of the optimal allocations of labor, produced goods, money demand, prices and policy instruments such that:

- The representative household maximizes the utility subject to the budget constraint taking prices and wages as given.
- Firms maximize their profit subject to the demand constraint taking prices and wages as given.
- Prices and wages are set in a way that all markets clear.
- The government maximizes the ex-ante expected welfare given the optimal actions of firms and the representative household.

Now let us find the optimal actions by different agents to characterize the equilibrium.

Households:

The representative household maximizes the utility¹³

$$U = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta}$$

¹¹The assumptions about values of parameters α_0 and β_0 guarantees the existence of the moment generating functions. Moreover, with this parametric assumption firms will *not* have any form of dispersed or asymmetric information after observing realizations of a_i , in another word there is no learning after observing a_i .

¹²The household always observes σ^2 .

¹³Given the symmetry in the environment and the log-normal assumption, the welfare is well defined in the closed form. Please find its expression in the appendix.

subject to the budget constraint

$$\int_I p_i c_i di + M = \int_I \Pi_i di + \int_I (1 - \tau) w_i n_i di + T.$$

Following the standard optimization problem that is presented in the appendix, we will obtain the consumption basket as:

$$\frac{p_i}{P} = \left(\frac{c_i}{C} \right)^{-\frac{1}{\rho}} \quad (1)$$

which is standard given the CES assumption. Moreover, the labour supply (labor basket) is given by:

$$\frac{(1 - \tau) w_i}{W} = \left(\frac{n_i}{N} \right)^{-\frac{1}{\omega}} \quad (2)$$

The second order condition holds as long as $\omega < 0$ which implies that higher labour will be allocated to the firm with higher relative wage.

Finally the optimal money demand will be:

$$\left(\frac{M}{P} \right)^{-\delta} = C^{-\gamma} \quad (3)$$

Firms:

Firms maximize their profit π_i subject to the consumption basket (1). In the absence of information friction the optimal price setting will be:

$$p_i = \frac{\rho}{\rho - 1} \frac{w_i}{A_i} \quad (4)$$

which is standard price setting equation and implies the optimal price is the mark-up $\frac{\rho}{\rho-1}$ multiplied by the marginal cost $\frac{w_i}{A_i}$.

Government:

As it is shown in the appendix, in the baseline model without information friction, real variables are pinned down regardless of the conduct of the monetary policy so the monetary policy is neutral and does not have any real effect. Therefore, the government only chooses the optimal tax rate τ such that given the optimal decisions by households and firms, the expected welfare is maximized. The optimal fiscal policy eliminates monopolistic competition distortion.

Equilibrium conditions in closed form are provided in the appendix. As you see, given the log-normal distribution and symmetric assumptions, all real and nominal variable allocations are log-linear in terms of states.

3 Contractionary dispersion shock

As it was mentioned before the idiosyncratic productivity is log-normally distributed:

$$a_i = \log(A_i) \quad \sim \quad N \left(\bar{a} - \frac{\sigma^2}{2}, \sigma^2 \right).$$

Given $\bar{a}$ and σ^2 the expected value of the productivity will be:

$$E(A_i) = E(e^{a_i}) = e^{\bar{a}}$$

Therefore, $\bar{a}$ implies the average value of the productivity in the economy. Assumed distribution of productivities guarantees that σ^2 pins down the degree of productivity dispersion in the economy without affecting the average value.

I study the effect of an increase in the productivity dispersion on the economy. Does higher dispersion in the productivity results in higher output and employment or not?

As it is shown in the Appendix, the aggregate output can be expressed in terms of the average aggregate TFP $\bar{a}$ and the TFP dispersion σ^2 :

$$\log(Y) = \log(C) = C_0 + C_A \bar{a} + C_\sigma \sigma^2$$

$$C_A = \frac{1 - \epsilon}{\gamma - \epsilon} \quad , \quad C_\sigma = \frac{-(1 - \epsilon)[1 + \omega(\rho - 2)]}{2(\rho - \omega)(\gamma - \epsilon)}$$

C_A is always positive. In another word, higher average TFP implies higher output. However, based on the model parameters C_σ can be positive (expansionary dispersion shock) or negative (contractionary dispersion shock).

For small values of the elasticity of substitution of labor (ω close to zero) or small values of the elasticity of substitution of consumed goods (ρ close to 1), the coefficient C_σ will be negative and we will obtain contractionary dispersion shocks. The exact threshold values for these elasticities are provided in the appendix. Contractionary dispersion shock is a novel finding and is absent in standard friction-less models because of the well-known Oi-Hartman-Abel effect.

To understand the main intuition for this result let us study the relative labor supply in two different firms i and j :

$$\log(n_i) - \log(n_j) = \frac{-\omega(\rho - 1)}{\rho - \omega} (a_i - a_j)$$

Therefore, due to the complementarity effect, labor will be allocated to a more productive firm given the fact that $\omega < 0$ and $\rho > 1$. Let us assume $|\omega|$ converges to zero.¹⁴ It means that we are converging to the Leontief environment in which the labour supply in all firms are constant and equal to the aggregate labor supply. By this assumption basically we are eliminating the complementarity effect between labor and productivity.

By assuming small values for elasticity of substitution of labor $|\omega|$ and large values for elasticity of substitution of consumption good ρ , the wage aggregator equation will converge to simple uniform integration while the price aggregator will be same as before:

$$P = \left[\int_I (p_i)^{1-\rho} di \right]^{\frac{1}{1-\rho}} \quad , \quad W = (1 - \tau) \int_I w_i di$$

So given the mean preserving distribution of productivity and the law of large numbers, the responsiveness of aggregate wage to the dispersion shock is equal to the responsiveness of idiosyncratic wage to the dispersion shock. However, the responsiveness of aggregate price to the dispersion shock is *larger* than the responsiveness of idiosyncratic price to the dispersion shock. Moreover, the optimal price setting condition (4) implies that the responsiveness of idiosyncratic price to the dispersion shock is equal to the responsiveness of idiosyncratic wage to the dispersion shock. Therefore, after a positive dispersion shock, the aggregate real wage $\frac{W}{P}$ will decrease, which results in lower aggregate consumption and higher aggregate labor supply.

¹⁴The intuition for small values of ρ is similar.

4 Model with nominal rigidity

4.1 Environment and equilibrium

Firms can set the price optimally if they have complete information about the aggregate states which affect their marginal cost of production. Now, I introduce the information friction as a source of nominal rigidity. In particular, I assume firms have incomplete information about the productivity dispersion σ^2 when they set the price¹⁵. Prices are chosen based on firms' expectation about σ^2 and can not be updated afterward. In contrast to Angeletos et al. (2020) and Angeletos and La'O (2020) firms have complete information about the aggregate productivity $\bar{a}$ and the only source of uncertainty is the productivity dispersion σ^2 . All firms share the same prior about the unknown dispersion. In particular, it is assumed that the dispersion is drawn from the following known distribution, but its true realization is unknown for firms:

$$\sigma^2 \sim \text{Inverse} - \text{Gamma}(\alpha_0, \beta_0) \quad s.t. \quad \begin{cases} \beta_0 \rightarrow \infty, \\ \alpha_0 \rightarrow \infty, \\ \frac{\beta_0}{\alpha_0} \rightarrow \sigma_0^2 \end{cases}$$

Parametric assumptions above guarantee the existence of moment generating function for the unconditional distribution of idiosyncratic productivities. Moreover, with this parametric assumption, firms will *not* have any form of asymmetric or dispersed information after observing the realization of idiosyncratic productivity a_i , in another word they will not learn from idiosyncratic productivity a_i .

So how do information friction and nominal rigidity affect the equilibrium conditions? Please note all assumptions are similar to the baseline model that is presented in the previous section and the only departure from the baseline model is the introduction of the information friction about productivity dispersion. Therefore, the only condition that is different from the baseline model is the price setting condition. As firms have incomplete information about the aggregate state, the objective of a firm is to maximize its *expectation* of the representative consumer's valuation of its profit subject to the consumption basket (1), namely:

$$\max_{p_i, n_i} E \left[\frac{U'(C)}{P} (p_i y_i - w_i n_i) \right] \quad s.t. \quad \frac{p_i}{P} = \left(\frac{c_i}{C} \right)^{-\frac{1}{\rho}}$$

As it is shown in the appendix the profit maximization results in the following price setting condition:

$$p_i = \frac{\rho}{A_i(\rho - 1)} \frac{E \left[\frac{U'(C)}{P} w_i n_i \right]}{E \left[\frac{U'(C)}{P} n_i \right]} = \frac{\rho}{A_i(\rho - 1)} \left(\frac{\text{Cov} \left[\frac{U'(C)}{P} n_i, w_i \right]}{E \left[\frac{U'(C)}{P} n_i \right]} + E(w_i) \right) \quad (5)$$

This equation is equivalent to the equation (4) with the only difference that instead of the true realization of the marginal cost w_i , we have new expectation terms which is referring to the covariance channel between the marginal cost of production and the factor of production (*risk channel*) in addition to the expected marginal cost. You can easily see if the wage does not depend on the unknown productivity dispersion, equations (4) and

¹⁵However, the household has complete information about the productivity dispersion.

(5) will be identical. We will come back to this in the next section when we study the optimal monetary policy.

In order to find the equilibrium conditions in the model with information friction, I use the standard guess and verify approach. Because of the symmetry in the environment and log-normal assumption, it is easy to show that all variables are log-linear in terms of the known states. In particular, I guess following policy functions for the household, firms and the government:

- Household's policy functions:

$$- \log(n_i) = n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2. \quad ^{16}$$

$$- \log(N) = N_0 + N_A \bar{a} + N_\sigma \sigma^2.$$

$$- \log(C) = \log(Y) = C_0 + C_A \bar{a} + C_\sigma \sigma^2.$$

- Firms' policy functions:

$$- \log(p_i) = \psi_0 + \psi_a a_i + \psi_A \bar{a}.$$

$$- \log(P) = P_0 + P_A \bar{a} + P_\sigma \sigma^2.$$

- Fiscal Policy: $\log(1 - \tau) = \tau_0$.

- Monetary Policy: $\log(M) = m_\sigma \sigma^2$.

and using equilibrium conditions verify that my log-linear guess is valid. The values of 16 unknown agents' policy functions' coefficients (excluding fiscal and monetary policies) are determined and presented in the appendix. m_σ and τ_0 are policy tools in the control of the government. Same as before, τ_0 is chosen optimally such that the monopolistic competition distortion will be eliminated. Optimal monetary policy will be determined in the next section.

Please note that the household has complete information about aggregate dispersion when makes the decision about labor supply. That is why the dispersion appears in the labor supply policy function but not in the idiosyncratic price function. Moreover, based on the law of large numbers the aggregate values for labor, consumption and price are functions of the productivity dispersion in the market.

Monetary policy's response to known states such as aggregate average productivity $\bar{a}$ does not have any real effect in the economy. In another word, as it was discussed in the baseline model, the monetary policy response to known states is neutral. Therefore, I assume the monetary policy rule $\log(M) = m_\sigma \sigma^2$ which only responds to the unknown dispersion. This policy has real effect on the economy and is not neutral anymore. Specifically, coefficients n_σ , C_σ and N_σ in policy functions depend on the monetary policy rule m_σ .

¹⁶I only consider the labor supply in each firm, because due to the market clearing labor supply and labor demand in each firm are equal.

5 Optimal monetary policy

As it is discussed in previous sections, monetary policy in the environment without information friction is neutral and only affects nominal variables. However, in order to understand the optimal monetary policy in the environment with information friction, it is useful to study the impact of the monetary policy on nominal variables in the frictionless model without the information friction. In particular, let us study the effect of the monetary policy on firms' idiosyncratic nominal price and wage in the model without information friction:

$$\log(p_i) = \psi'_0 + \psi'_a a_i + \underbrace{(m_\sigma - Q_s)}_{\psi'_\sigma} \sigma^2 + \psi'_A \bar{a} = \log(w_i) - a_i + \log\left(\frac{\rho}{\rho - 1}\right)$$

where ψ'_0 , ψ'_a , ψ'_A and Q_s are constant functions of model parameters. You can find their values in the appendix and the equation (6).

Consider a specific monetary policy such that $m_\sigma = Q_s$ which results in $\psi_\sigma = 0$. In another word, for $m_\sigma = Q_s$ neither idiosyncratic nominal price nor idiosyncratic nominal wage respond to the productivity dispersion. This policy eliminates the dependence of idiosyncratic nominal variables on the productivity dispersion in the full information environment.

Now, let us go to the environment with information friction, in which the monetary policy is not neutral anymore. In order to determine the optimal conduct of monetary policy here, we need to understand what the source of friction in the market is and how the monetary policy can eliminate it. There are two sources of frictions in this environment; the first one is the monopolistic competition distortion and the second one is the information friction. The monopolistic competition distortion can be eliminated using the optimal fiscal policy. So we only have one source of friction left and that is the information driven nominal rigidity.

Monetary policy can easily remove this friction. As we saw before, the information friction only affects the firms' price setting problem. So by removing the dependence of idiosyncratic nominal variables on the productivity dispersion, the information friction becomes irrelevant. This policy is exactly equivalent to set $m_\sigma = Q_s$. By doing so, the monetary policy eliminates the relevance of productivity dispersion both in the full information model and in the model without information friction. Please note that sign of Q_s is not predetermined and depends on many parameters that affects the cyclicity of the dispersion shock and convexity of the welfare. Therefore, we can not conclude that optimal monetary policy's response to the dispersion shock is necessarily expansionary or contractionary.

As mentioned above, the optimal monetary policy is the policy that makes the information friction irrelevant. However, this policy *is not an inflation targeting policy*. In particular, the optimal monetary policy is the policy that stabilize the idiosyncratic prices p_i . In this environment stabilizing idiosyncratic prices does not result in stabilizing the aggregate price. Following the law of large number by aggregating idiosyncratic prices using the price aggregator, and because of the fact that idiosyncratic prices are functions of idiosyncratic productivities a_i , the aggregate price P will still depend on the productivity dispersion σ^2 , although idiosyncratic prices do not depend on the productivity dispersion σ^2 . In other words, although through optimal monetary policy we make idiosyncratic prices irrelevant of σ^2 by making $\psi'_\sigma = 0$, because $\psi'_a \neq 0$ after aggregating idiosyncratic prices following the law of large number, the aggregate price will still depend on the

unknown dispersion shock σ^2 . Please also note that optimal monetary policy does *not* depend on the agents' prior beliefs.

In the figure 1 you see the ex-ante expected welfare for a given parameterization of the model. As you notice, in the model with information friction for the monetary policy $m_\sigma = Q_s$, the expected ex-ante welfare is maximized and will be equal to the expected welfare in the full-information *flexible price* scenario.

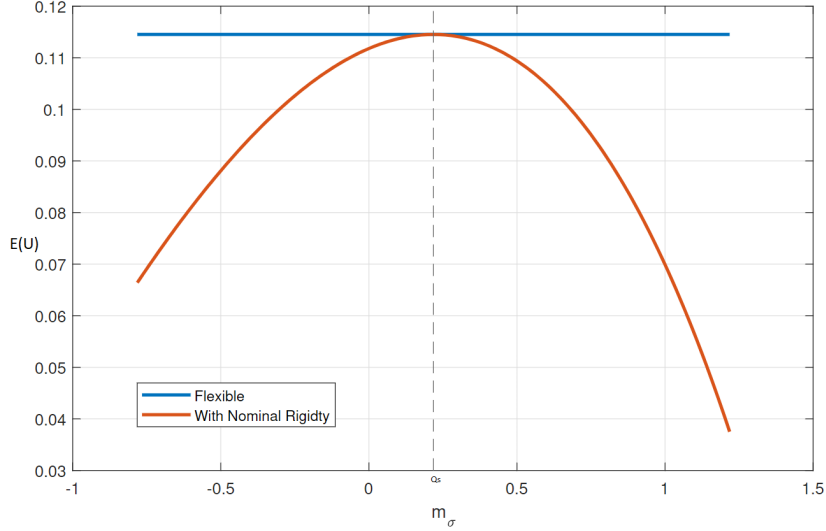


Figure 1: Expected welfare in response to the monetary policy in models with information friction and without information friction

6 Conclusion

In this paper I studied the impact of the productivity dispersion shock. By introducing taste for variety and substitutability, not only for the consumption good but also for the supplied labor, I managed to obtain the contractionary productivity dispersion shock in a fiction-less environment. The contractionary second moment shock, which is a departure from the well known Oi-Hartman-Abel effect, is a novel result and is derived when the elasticity of substitution either for the labor supply or consumption good is small enough. The intuition is very simple; by reducing the elasticity of substitution we converge to the Leontief environment such that the complementarity channel between productivity and factors of production is broken.

In order to study the impact of the dispersion shock on the conduct of the monetary policy, I introduced information friction as a source of nominal rigidity following Angeletos and La'O (2020), Angeletos et al. (2020) and La'O and Tahbaz-Salehi (2022). In particular, I assumed firms have incomplete information about the productivity dispersion when they set prices of their goods.

I showed the optimal monetary policy is the policy that eliminates the dependence of the idiosyncratic nominal variables on the productivity dispersion. This policy basically replicates the flexible price full information equilibrium, however, it is not an inflation targeting policy. My result is in contrast to the well-known “Paradox of Transparency” literature.

A Appendix 1

A.1 Baseline model without nominal rigidity

In this section after summarizing the model without nominal rigidity, the optimal conditions will be derived:

A.1.1 Environment:

Household:

- Utility:

$$U = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta}$$

- Aggregation:

$$C = \left[\int_I (c_i)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}, \quad P = \left[\int_I (p_i)^{1-\rho} di \right]^{\frac{1}{1-\rho}}$$

$$N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di \right]^{\frac{\omega}{\omega-1}}, \quad W = \left[\int_I [(1-\tau)w_i]^{1-\omega} di \right]^{\frac{1}{1-\omega}}$$

- Household's Budget Constraint:

$$\int_I p_i c_i di + M = \int_I \Pi_i di + \int_I (1-\tau)w_i n_i di + T$$

- Government's Budget Constraint:

$$T = \int_I \tau w_i n_i di + M$$

Firms:

- Monopolistic Competitive Firms producing differentiated goods:

$$y_i = A_i n_i$$

- Profit Maximization Problem:

$$\max_{p_i, n_i} p_i y_i - w_i n_i \quad s.t. \quad \frac{p_i}{P} = \left(\frac{\widehat{A_i n_i}}{C} \right)^{-\frac{1}{\rho}}$$

$$\max_{p_i, n_i} C (p_i)^{1-\rho} P^\rho - w_i n_i \quad s.t. \quad \frac{p_i}{P} = \left(\frac{\widehat{A_i n_i}}{C} \right)^{-\frac{1}{\rho}}$$

A.1.2 Optimal Conditions:

I summarize the main optimal conditions here. The details of the optimization are provided in the appendix 2.

- Consumption basket:

$$\frac{p_i}{P} = \left(\frac{c_i}{C} \right)^{\frac{-1}{\rho}}$$

- Labour supply:

$$\frac{(1 - \tau)w_i}{W} = \left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}} \quad or \quad \frac{P}{W} = \frac{C^{-\gamma}}{N^{-\epsilon}}$$

- Money demand:

$$\left(\frac{M}{P} \right)^{-\delta} = C^{-\gamma}$$

- Price setting :

$$p_i = \frac{\rho}{\rho - 1} \frac{w_i}{A_i}$$

A.1.3 Equilibrium:

Using above equations we can easily find the equilibrium in a model without the nominal rigidity ¹⁷. Start from the consumption basket equation. From the market clearing and after some rearrangement we will have:

$$\frac{p_i^{1-\rho}}{P^{-\rho}} C = p_i A_i n_i \rightarrow \text{Integration} \rightarrow PC = \int p_i A_i n_i di \rightarrow \text{Price Setting} \rightarrow PC = \frac{\rho}{\rho - 1} \int w_i n_i di$$

In the same way we will obtain the following condition from the labour basket:

$$NW = (1 - \tau) \int w_i n_i di$$

So:

$$\frac{PC}{WN} = \frac{\rho}{(\rho - 1)(1 - \tau)} \rightarrow \text{Labour Supply} \rightarrow \frac{C^{1-\gamma}}{N^{1-\epsilon}} = \frac{\rho}{(\rho - 1)(1 - \tau)}$$

Next, by dividing the consumption basket by labor supply we will have:

$$\begin{aligned} \frac{p_i}{w_i} \frac{W}{P(1 - \tau)} &= \frac{\left(\frac{c_i}{C} \right)^{\frac{-1}{\rho}}}{\left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}}} \rightarrow \frac{\rho}{(\rho - 1)A_i} \frac{N^{1-\epsilon}}{C^{1-\gamma}(1 - \tau)} = \frac{\left(\frac{c_i}{C} \right)^{\frac{-1}{\rho}}}{\left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}}} \frac{N}{C} \rightarrow \frac{(A_i n_i)^{\frac{1}{\rho}}}{A_i n_i^{\frac{1}{\omega}}} = \frac{C^{\frac{1-\rho}{\rho}}}{N^{\frac{1-\omega}{\omega}}} \\ A_i^{\frac{1-\rho}{\rho}} n_i^{\frac{\omega-\rho}{\rho\omega}} &= \frac{C^{\frac{1-\rho}{\rho}}}{N^{\frac{1-\omega}{\omega}}} \rightarrow n_i = \frac{C^{\frac{\omega(1-\rho)}{\omega-\rho}}}{N^{\frac{\rho(1-\omega)}{\omega-\rho}}} A_i^{\frac{\omega(\rho-1)}{\omega-\rho}} \end{aligned}$$

¹⁷Guess and verify is the standard approach in this literature to find the equilibrium. However, in the friction-less model for a better illustration, I find the equilibrium directly without using guess and verify method. I use this approach later in the model with nominal rigidity.

We know $N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di \right]^{\frac{\omega}{\omega-1}}$ so by integrating n_i and after some rearrangement we will have:

$$\left[\frac{N}{C} \right]^{\frac{(\omega-1)(1-\rho)}{\omega-\rho}} = \int_I A_i^{\frac{(\omega-1)(\rho-1)}{\omega-\rho}} di$$

Using the law of large number and the mentioned distribution for A_i we will see:

$$\int_I A_i^{\frac{(\omega-1)(\rho-1)}{\omega-\rho}} di = e^{\frac{(\omega-1)(\rho-1)}{\omega-\rho} \left(\bar{a} - \frac{\sigma^2}{2} \right) + \frac{(\omega-1)^2(\rho-1)^2\sigma^2}{2(\omega-\rho)^2}}$$

So

$$\frac{C}{N} = e^{\bar{a} + \left[\frac{(\omega-1)(\rho-1)}{\omega-\rho} - 1 \right] \frac{\sigma^2}{2}}$$

We have already found:

$$\frac{PC}{WN} = \frac{C^{1-\gamma}}{N^{1-\epsilon}} = \frac{\rho}{(\rho-1)(1-\tau)}$$

So

$$e^{(1-\gamma)\bar{a} + \left[\frac{(\omega-1)(\rho-1)}{\omega-\rho} - 1 \right] \frac{(1-\gamma)\sigma^2}{2}} N^{\epsilon-\gamma} = \frac{\rho}{(\rho-1)(1-\tau)} \rightarrow$$

$$\log(N) = \underbrace{\frac{\log\left(\frac{\rho}{(\rho-1)(1-\tau)}\right)}{\epsilon-\gamma}}_{N_0} + \underbrace{\frac{\gamma-1}{\epsilon-\gamma}\bar{a}}_{N_A} + \underbrace{\frac{(\gamma-1)[1+\omega(\rho-2)]}{2(\omega-\rho)(\epsilon-\gamma)}\sigma^2}_{N_\sigma}$$

And we can easily find $\log(C)$:

$$\log(C) = \underbrace{\frac{\log\left(\frac{\rho}{(\rho-1)(1-\tau)}\right)}{\epsilon-\gamma}}_{C_0} + \underbrace{\frac{\epsilon-1}{\epsilon-\gamma}\bar{a}}_{C_A} + \underbrace{\frac{(\epsilon-1)[1+\omega(\rho-2)]}{2(\omega-\rho)(\epsilon-\gamma)}\sigma^2}_{C_\sigma}$$

After finding aggregate real variables we can easily find the idiosyncratic real variables:

$$n_i = \frac{C^{\frac{\omega(1-\rho)}{\omega-\rho}}}{N^{\frac{\rho(1-\omega)}{\omega-\rho}}} A_i^{\frac{\omega(\rho-1)}{\omega-\rho}} \rightarrow$$

$$\begin{aligned} \log(n_i) = & \frac{\omega(1-\rho)}{\omega-\rho} C_0 - \frac{\rho(1-\omega)}{\omega-\rho} N_0 + \left(\frac{\omega(1-\rho)}{\omega-\rho} C_A - \frac{\rho(1-\omega)}{\omega-\rho} N_A \right) \bar{a} + \\ & \left(\frac{\omega(1-\rho)}{\omega-\rho} C_\sigma - \frac{\rho(1-\omega)}{\omega-\rho} N_\sigma \right) \sigma^2 + \frac{\omega(\rho-1)}{\omega-\rho} a_i \end{aligned}$$

and $\log(c_i) = a_i + \log(n_i)$. As you can see all real variables are determined regardless of the monetary policy. Therefore, in the framework without nominal rigidity *the monetary policy is neutral*.

Regarding the nominal variables, from the money demand equation you can easily see that the aggregate price is pinned down by the monetary policy and aggregate output $P = \frac{M}{C^{\frac{1}{\gamma}}}$ and by replacing it the consumption basket we can easily find the relationship between nominal wages (both the idiosyncratic and the aggregate) and the monetary policy and real variables. Finally from the price setting equation, by replacing the idiosyncratic

wage, you can easily find the relationship between idiosyncratic price and the monetary policy and real variables:

$$\log(p_i) = \underbrace{\frac{-\gamma C_0}{\delta}}_{\psi'_0} + \underbrace{\frac{1-\omega}{\omega-\rho}}_{\psi'_a} a_i + \underbrace{\left(m_\sigma - \left[\frac{(1-\rho)\psi'_a{}^2}{2} + \frac{\gamma C_\sigma}{\delta} - \frac{\psi'_a}{2} \right] \right)}_{\psi'_\sigma} \sigma^2 + \underbrace{\left(-\psi_a - \frac{\gamma C_A}{\delta} \right)}_{\psi'_A} \bar{a} \quad (6)$$

$$\log(w_i) = \log(p_i) + a_i - \log\left(\frac{\rho}{\rho-1}\right)$$

I assume the monetary policy follows the rule $\log(M) = m_\sigma \sigma^2$. The intuition for this policy rule is provided in the section of the model with nominal rigidity. As you can see ψ'_a is always negative which means more productive firms set lower prices.

Now let us go to *the optimal fiscal policy*. After finding the equilibrium allocations, we can express the ex-ante expected welfare of the government, before realization of the shock as follows:

$$E(U) = E\left(\frac{C^{1-\gamma}-1}{1-\gamma} - \frac{N^{1-\epsilon}-1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta}-1}{1-\delta}\right) =$$

$$E\left(\frac{e^{(1-\gamma)(C_0+C_A\bar{a}+C_\sigma\sigma_0^2)}-1}{1-\gamma} - \frac{e^{(1-\epsilon)(N_0+N_A\bar{a}+N_\sigma\sigma_0^2)}-1}{1-\epsilon} + \frac{e^{(1-\delta)[\frac{\gamma C_0}{\delta}+\frac{\gamma C_A}{\delta}\bar{a}+\frac{\gamma C_\sigma}{\delta}\sigma_0^2]}-1}{1-\delta}\right) =$$

$$\frac{e^{(1-\gamma)(C_0+C_A\bar{a}+C_\sigma\sigma_0^2)}-1}{1-\gamma} - \frac{e^{(1-\epsilon)(N_0+N_A\bar{a}+N_\sigma\sigma_0^2)}-1}{1-\epsilon} + \frac{e^{(1-\delta)[\frac{\gamma C_0}{\delta}+\frac{\gamma C_A}{\delta}\bar{a}+\frac{\gamma C_\sigma}{\delta}\sigma_0^2]}-1}{1-\delta}$$

I use the standard Normal-Gamma prior. It is well-known in the literature that if the conditional distribution of the variable x is normally distributed $x|\sigma^2 \sim N(\mu, \sigma^2)$ and $\sigma^2 \sim \text{Inverse-Gamma}(\alpha_0, \beta_0)$, then the unconditional distribution of the variable x is not normal anymore and will be distributed according to the fatter tail student's t distribution $x \sim t_{2\alpha_0}(\mu, \sigma_0^2)$. We know that the moment generating function for the student's t distribution does not exist, so in general the expected welfare above is not well defined. However, by assuming the parametric assumption that $\alpha_0 \rightarrow \infty$ and $\beta_0 \rightarrow \infty$, the degree of freedom for the student's t distribution goes to infinity and we converge to the normal distribution. Moreover, as $\frac{\beta_0}{\alpha_0} \rightarrow \sigma_0^2$, the expected welfare converges to a finite value.

By assuming $\log(1-\tau) = \tau_0$, we can express the optimal fiscal policy as:

$$\partial E(U)/\partial \tau_0 = 0 \quad \rightarrow \quad e^{(1-\gamma)(C_0+C_A\bar{a}+C_\sigma\sigma_0^2)} + e^{\frac{\gamma(1-\delta)}{\delta}(C_0+C_A\bar{a}+C_\sigma\sigma_0^2)} \frac{\gamma}{\delta} = e^{(1-\epsilon)(N_0+N_A\bar{a}+N_\sigma\sigma_0^2)}$$

And after replacing equilibrium values for policy functions' coefficients we can *numerically* find the optimal fiscal policy which eliminates the monopolistic competition distortion.

A.1.4 Contractionary Dispersion Shock

From the previous subsection we know $\log(C) = \log(Y) = C_0 + C_A\bar{a} + C_\sigma\sigma^2$. It is easy to see that C_A is always greater than zero which implies higher average TFP will increase

the output. How about C_σ ?

$$C_\sigma = -\frac{(1-\epsilon)[1+\omega(\rho-2)]}{2(\rho-\omega)(\gamma-\epsilon)}$$

We know $\epsilon < 0$, $\rho > 1 > 0 > \omega$ and $\gamma > 0 > \epsilon$. Therefore, you can easily see that there exists a value $\omega^* = \frac{-1}{\rho-2}$, by assuming $\rho > 2$, such that for $\omega^* < \omega < 0$ we will have $C_\sigma < 0$ or in another word, contractionary dispersion shock. In the same way, there exists a value $\rho^* = 2 - \frac{1}{\omega}$ such that for $1 < \rho < \rho^*$ the coefficient C_σ will be negative. Either of these two conditions imply that we need a small degree of substitution for labor or consumed goods to be able to have a contractionary dispersion shock.

A.2 Model with nominal rigidity

A.2.1 Optimal Price Setting Condition

Let us study the profit maximization problem of a firm:

$$\max_{p_i, n_i} E \left[\frac{U'(C)}{P} (p_i y_i - w_i n_i) \right] \quad s.t. \quad \frac{p_i}{P} = \left(\frac{c_i}{C} \right)^{-\frac{1}{\rho}}$$

After replacing the production function $c_i = y_i = A_i n_i$ in the consumption basket we will have:

$$n_i = \left(\frac{C P^\rho}{A_i} \right) (p_i)^{-\rho}$$

And then plugging it into the firms problem:

$$\begin{aligned} E \left[\frac{U'(C)}{P} (p_i A_i \quad \partial n_i / \partial p_i + A_i n_i - w_i \quad \partial n_i / \partial p_i) \right] &= 0 \\ E \left[\frac{U'(C)}{P} \left(-\rho p_i A_i \frac{n_i}{p_i} + A_i n_i + \rho w_i \frac{n_i}{p_i} \right) \right] &= 0 \\ p_i &= \frac{\rho}{A_i(\rho-1)} \frac{E \left[\frac{U'(C)}{P} w_i n_i \right]}{E \left[\frac{U'(C)}{P} n_i \right]} \end{aligned}$$

A.2.2 Equilibrium

I use guess and verify approach to determine equilibrium conditions. I guess following policy functions for agents and the government:

- Household's policy functions:
 - $\log(n_i) = n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2$.
 - $\log(N) = N_0 + N_A \bar{a} + N_\sigma \sigma^2$.
 - $\log(C) = \log(Y) = C_0 + C_A \bar{a} + C_\sigma \sigma^2$.
- Firms' policy functions:
 - $\log(p_i) = \psi_0 + \psi_a a_i + \psi_A \bar{a}$.

$$- \log(P) = P_0 + P_A \bar{a} + P_\sigma \sigma^2.$$

- Fiscal Policy: $\log(1 - \tau) = \tau_0$.
- Monetary Policy: $\log(M) = m_\sigma \sigma^2$.

and plug these functions in the aggregation and optimal conditions using the market clearing in order to determine 16 unknown coefficients (excluding policies' coefficients):

- Aggregate consumption:

$$\begin{aligned} e^{C_0 + C_A \bar{a} + C_\sigma \sigma^2} &= \left[\int_I (y_i)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \rightarrow \text{Market clearing} \rightarrow = \\ &= \left[\int_I \left(e^{a_i + (n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2)} \right)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} = \left[\int_I \left(e^{\frac{\rho-1}{\rho} [a_i + (n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2)]} \right) di \right]^{\frac{\rho}{\rho-1}} = \\ &= e^{[n_0 + n_A \bar{a} + n_\sigma \sigma^2]} \left[\int_I \left(e^{\frac{(\rho-1)a_i}{\rho} [1 + n_a]} \right) di \right]^{\frac{\rho}{\rho-1}} \rightarrow \text{Law of large numbers} \rightarrow = \\ &= e^{[n_0 + n_A \bar{a} + n_\sigma \sigma^2]} e^{[1 + n_a] \bar{a} + \left[\frac{[1 + n_a]^2 (\rho-1)}{\rho} - 1 - n_a \right] \frac{\sigma^2}{2}} \end{aligned}$$

After matching coefficients we will have:

$$C_A = n_a + n_A + 1 \quad , \quad C_\sigma = n_\sigma + \frac{(\rho-1)[1 + n_a]^2}{2\rho} - \frac{1 + n_a}{2} \quad , \quad C_0 = n_0$$

- Aggregate labor:

$$\begin{aligned} e^{N_0 + N_A \bar{a} + N_\sigma \sigma^2} &= \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di \right]^{\frac{\omega}{\omega-1}} = \left[\int_I \left(e^{\frac{\omega-1}{\omega} [n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2]} \right) di \right]^{\frac{\omega}{\omega-1}} = \\ &= e^{n_0 + n_A \bar{a} + n_\sigma \sigma^2} \left[\int_I \left(e^{\frac{(\omega-1)n_a a_i}{\omega}} \right) di \right]^{\frac{\omega}{\omega-1}} = e^{n_0 + n_A \bar{a} + n_\sigma \sigma^2} e^{n_a \bar{a} + \left[\frac{n_a^2 (\omega-1)}{\omega} - n_a \right] \frac{\sigma^2}{2}} \end{aligned}$$

After matching coefficients we will have:

$$N_A = n_a + n_A \quad , \quad N_\sigma = n_\sigma + \frac{(\omega-1)n_a^2}{2\omega} - \frac{n_a}{2} \quad , \quad N_0 = n_0$$

- Aggregate price:

$$\begin{aligned} e^{P_0 + P_A \bar{a} + P_\sigma \sigma^2} &= \left[\int_I (p_i)^{1-\rho} di \right]^{\frac{1}{1-\rho}} = \left[\int_I e^{(1-\rho)(\psi_0 + \psi_a a_i + \psi_A \bar{a})} di \right]^{\frac{1}{1-\rho}} = e^{\psi_0 + \psi_A \bar{a}} \left[\int_I e^{(1-\rho)[\psi_a a_i]} di \right]^{\frac{1}{1-\rho}} \\ &= e^{\psi_0 + \psi_A \bar{a}} e^{\psi_a \bar{a} + \left[(1-\rho)\psi_a^2 - \psi_a \right] \frac{\sigma^2}{2}} \end{aligned}$$

After matching coefficients we will have:

$$P_A = \psi_a + \psi_A \quad , \quad P_\sigma = \frac{(1-\rho)\psi_a^2}{2} - \frac{\psi_a}{2} \quad , \quad P_0 = \psi_0$$

- Money demand:

$$e^{\delta P_0 + \delta P_A \bar{a} - \delta(m_\sigma - P_\sigma)\sigma^2} = e^{-\gamma(C_0 + C_A \bar{a} + C_\sigma \sigma^2)}$$

After matching coefficients we will have:

$$-\delta P_A = \gamma C_A \quad , \quad \delta(m_\sigma - P_\sigma) = \gamma C_\sigma \quad , \quad -\delta P_0 = \gamma C_0$$

- Consumption basket:

$$-\rho \left(\underbrace{\log(p_i) - \log(P)}_{\psi_a(a_i - \bar{a}) - [(1-\rho)\psi_a^2 - \psi_a]\frac{\sigma^2}{2}} \right) = \log(y_i) - \log(Y)$$

$$\begin{aligned} \log(y_i) - \log(Y) &\rightarrow \text{Market Clearing} \rightarrow = \log(A_i) + (\log(n_i)) - \log(Y) = \\ &= a_i + [n_0 + n_a a_i + n_A \bar{a} + n_\sigma^2] - (C_0 + C_A \bar{a} + C_\sigma^2) = \\ &= [1 + n_a] a_i + [n_A - C_A] \bar{a} + [n_\sigma - C_\sigma] \sigma^2 + n_0 - C_0 \end{aligned}$$

Here after matching coefficients we will obtain only one new equation:

$$-\rho \psi_a = 1 + n_a$$

- Price Setting:

Using the consumption basket, labor basket and market clearing conditions after replacing the idiosyncratic wage w_i in the price setting condition you can easily see:

$$E \left(C^{1-\gamma} \left(\frac{p_i}{P} \right)^{1-\rho} \right) = \frac{\rho}{\rho-1} \quad E \left(N^{-\epsilon + \frac{1}{\omega}} \frac{n_i^{\frac{\omega-1}{\omega}}}{1-\tau} \right) = \frac{\rho}{\rho-1} \quad E \left(N^{1-\epsilon} \frac{\left(\frac{n_i}{N} \right)^{\frac{\omega-1}{\omega}}}{1-\tau} \right)$$

I first express the left hand side of the equation in terms of states:

$$\begin{aligned} E \left(C^{1-\gamma} \left(\frac{p_i}{P} \right)^{1-\rho} \right) &= E \left(e^{(1-\gamma)(C_0 + C_A \bar{a} + C_\sigma \sigma^2) + (1-\rho)(\psi_a(a_i - \bar{a}) - [(1-\rho)\psi_a^2 - \psi_a]\frac{\sigma^2}{2})} \right) = \\ &= e^{(1-\gamma)C_0 + (1-\rho)\psi_a a_i + [(1-\gamma)C_A - (1-\rho)\psi_a]\bar{a}} \quad E \left(e^{\left[(1-\gamma)C_\sigma - \frac{(1-\rho)^2 \psi_a^2}{2} + \frac{(1-\rho)\psi_a}{2} \right] \sigma^2} \right) \end{aligned}$$

Rearranging and taking log of the left hand side:

$$(1-\rho)\psi_a a_i + [(1-\gamma)C_A - (1-\rho)\psi_a]\bar{a} + (1-\gamma)C_0 + \left[(1-\gamma)C_\sigma - \frac{(1-\rho)^2 \psi_a^2}{2} + \frac{(1-\rho)\psi_a}{2} \right] \sigma_0^2$$

And then let us go to the the right hand side

$$\begin{aligned} \frac{\rho}{\rho-1} \quad E \left(N^{1-\epsilon} \frac{\left(\frac{n_i}{N} \right)^{\frac{\omega-1}{\omega}}}{1-\tau} \right) &= \\ e^{\log\left(\frac{\rho}{\rho-1}\right)} \quad E \left[e^{-\tau_0 + (1-\epsilon)[N_0 + N_A \bar{a} + N_\sigma \sigma^2] + \frac{\omega-1}{\omega} \left(n_a[a_i - \bar{a}] - \left[\frac{(\omega-1)n_a^2}{\omega} - n_a \right] \frac{\sigma^2}{2} \right)} \right] &= \\ e^{\log\left(\frac{\rho}{\rho-1}\right)} \quad e^{-\tau_0 + (1-\epsilon)N_0 + \frac{\omega-1}{\omega} n_a a_i + [(1-\epsilon)N_A - \frac{\omega-1}{\omega} n_a]\bar{a}} \quad E \left(e^{\left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2 n_a^2}{2\omega^2} + \frac{(\omega-1)n_a}{2\omega} \right] \sigma^2} \right) \end{aligned}$$

Rearranging and taking log of the right hand side:

$$\left[\frac{\omega-1}{\omega} n_a \right] a_i + \left[(1-\epsilon)N_A - \frac{\omega-1}{\omega} n_a \right] \bar{a} \\ + \log \left(\frac{\rho}{\rho-1} \right) - \tau_0 + (1-\epsilon)N_0 + \left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2 n_a^2}{2\omega^2} + \frac{(\omega-1)n_a}{2\omega} \right] \sigma_0^2$$

After matching coefficients we will have:

$$(1-\rho)\psi_a = \frac{\omega-1}{\omega} n_a \quad , \quad (1-\gamma)C_A - (1-\rho)\psi_a = (1-\epsilon)N_A - \frac{\omega-1}{\omega} n_a \\ , \quad (1-\gamma)C_0 + \left[(1-\gamma)C_\sigma - \frac{(1-\rho)^2 \psi_a^2}{2} + \frac{(1-\rho)\psi_a}{2} \right] \sigma_0^2 = \\ \log \left(\frac{\rho}{\rho-1} \right) - \tau_0 + (1-\epsilon)N_0 + \left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2 n_a^2}{2\omega^2} + \frac{(\omega-1)n_a}{2\omega} \right] \sigma_0^2$$

So we obtained 16 linear equations to determine 16 unknown coefficients. Using basic algebra we can easily solve this system and find the coefficients. I first remind you the policy functions:

- Household's policy functions:

$$- \log(N) = N_0 + N_A \bar{a} + N_\sigma \sigma^2. \\ - \log(C) = \log(Y) = C_0 + C_A \bar{a} + C_\sigma \sigma^2. \\ - \log(n_i) = n_0 + n_a a_i + n_A \bar{a} + n_\sigma \sigma^2.$$

- Firms' policy functions:

$$- \log(p_i) = \psi_0 + \psi_a a_i + \psi_A \bar{a}. \\ - \log(P) = P_0 + P_A \bar{a} + P_\sigma \sigma^2.$$

and then I present obtained coefficients:

- Aggregate consumption:

$$C_A = \frac{\epsilon-1}{\epsilon-\gamma} \quad , \quad C_\sigma = \frac{\delta \left[m_\sigma - \frac{(1-\rho)\psi_a^2}{2} + \frac{\psi_a}{2} \right]}{\gamma} \\ C_0 = \frac{1}{(\epsilon-\gamma)} \left[\log \left(\frac{\rho}{\rho-1} \right) - \tau_0 + \left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2 n_a^2}{2\omega^2} - (1-\gamma)C_\sigma + \frac{(1-\rho)^2 \psi_a^2}{2} \right] \sigma_0^2 \right]$$

- Aggregate labor:

$$N_A = C_A - 1 = \frac{\gamma-1}{\epsilon-\gamma} \quad , \quad N_\sigma = C_\sigma - \frac{(\rho-1)\rho\psi_a^2}{2} + \frac{\omega-1}{2\omega} n_a^2 + \frac{1}{2} \quad , \quad N_0 = C_0$$

- Idiosyncratic labor:

$$n_a = \frac{\omega(\rho-1)}{\omega-\rho} \quad , \quad n_A = N_A - n_a \quad , \quad n_\sigma = N_\sigma - \frac{(\omega-1)n_a^2}{2\omega} + \frac{n_a}{2} \quad , \quad n_0 = N_0 = C_0$$

- Idiosyncratic price:

$$\psi_a = \frac{1-\omega}{\omega-\rho} \quad , \quad \psi_A = -\psi_a - \frac{\gamma C_A}{\delta} \quad , \quad \psi_0 = P_0$$

- Aggregate price:

$$P_A = \psi_a + \psi_A \quad , \quad P_\sigma = \frac{(1-\rho)\psi_a^2}{2} - \frac{\psi_a}{2} \quad , \quad P_0 = \frac{-\gamma C_0}{\delta}$$

B Appendix 2

B.1 Optimization problems of agents

Household:

I use Lagrangian method to find the optimal decision for the household:

$$L = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta} - \lambda \left[\int_I p_i c_i di + M - \int_I \Pi_i di - \int_I (1-\tau) w_i n_i di - T \right]$$

First order conditions with respect to different control variables imply:

- $\frac{\partial L}{\partial c_i}$:

$$C^{-\gamma} \frac{\partial C}{\partial c_i} - \lambda p_i = C^{-\gamma} \left(\frac{c_i}{C} \right)^{\frac{-1}{\rho}} - \lambda p_i = 0$$

$$\text{Define:} \quad \int_I p_i c_i di = X_c$$

$$C^{-\gamma+\frac{1}{\rho}} c_i^{\frac{1-\frac{1}{\rho}}{\rho}} = \lambda c_i p_i \Rightarrow C^{-\gamma+\frac{1}{\rho}} \int_I c_i^{\frac{\rho-1}{\rho}} di = \lambda X_c \Rightarrow C^{-\gamma+\frac{1}{\rho}} C^{\frac{\rho-1}{\rho}} = \lambda X_c \Rightarrow \frac{C^{1-\gamma}}{X_c} = \lambda$$

$$C^{-\gamma+\frac{1}{\rho}} c_i^{\frac{-1}{\rho}} = \frac{C^{1-\gamma}}{X_c} p_i \Rightarrow c_i^{\frac{-1}{\rho}} = \frac{C^{\frac{\rho-1}{\rho}} p_i}{X_c} \Rightarrow p_i c_i = \frac{C^{1-\rho} p_i^{1-\rho}}{X_c^{-\rho}} \Rightarrow X_c = \frac{C^{1-\rho}}{X_c^{-\rho}} \int_I p_i^{1-\rho} di$$

$$X_c = C \left[\int_I p_i^{1-\rho} di \right]^{\frac{1}{1-\rho}} = CP \quad , \quad \lambda = \frac{C^{1-\gamma}}{X_c} = \frac{C^{-\gamma}}{P}$$

$$c_i^{\frac{-1}{\rho}} = \frac{C^{\frac{\rho-1}{\rho}} p_i}{PC} \Rightarrow \frac{p_i}{P} = \left(\frac{c_i}{C} \right)^{\frac{-1}{\rho}}$$

- $\frac{\partial L}{\partial n_i}$:

$$-N^{-\epsilon} \frac{\partial N}{\partial n_i} + \lambda(1-\tau)w_i = -N^{-\epsilon} \left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}} + \lambda(1-\tau)w_i = 0 \quad 18$$

$$\text{Define:} \quad \int_I (1-\tau)w_i n_i di = X_n$$

$$N^{-\epsilon+\frac{1}{\omega}} n_i^{\frac{1-\frac{1}{\omega}}{\omega}} = \lambda(1-\tau)w_i n_i \Rightarrow N^{-\epsilon+\frac{1}{\omega}} N^{\frac{\omega-1}{\omega}} = \lambda X_n \Rightarrow \lambda = \frac{N^{1-\epsilon}}{X_n}$$

¹⁸It is easy to see that the second order condition holds as long as $\omega < 0$.

$$N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{\frac{-1}{\omega}} = \frac{N^{1-\epsilon}}{X_n} (1-\tau)w_i \Rightarrow n_i^{\frac{-1}{\omega}} = \frac{N^{\frac{\omega-1}{\omega}} (1-\tau)w_i}{X_n} \Rightarrow$$

$$(1-\tau)w_i n_i = \frac{N^{1-\omega} [(1-\tau)w_i]^{1-\omega}}{X_n^{-\omega}}$$

$$X_n^{1-\omega} = N^{1-\omega} \int_I [(1-\tau)w_i]^{1-\omega} di \Rightarrow X_n = N \left(\overbrace{\int_I [(1-\tau)w_i]^{1-\omega} di}^W \right)^{\frac{1}{1-\omega}} = NW \quad , \quad \lambda = \frac{N^{-\epsilon}}{W}$$

$$n_i = \frac{N^{1-\omega} [(1-\tau)w_i]^{-\omega}}{(NW)^{-\omega}} \Rightarrow \Rightarrow \quad \frac{(1-\tau)w_i}{W} = \left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}}$$

$$N^{-\epsilon} \quad \left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}} = \frac{C^{-\gamma}}{P} (1-\tau)w_i \Rightarrow w_i = \frac{N^{-\epsilon} P}{C^{-\gamma} (1-\tau)} \quad \left(\frac{n_i}{N} \right)^{\frac{-1}{\omega}}$$

$$N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{\frac{-1}{\omega}} = \lambda (1-\tau)w_i = \frac{C^{-\gamma}}{P} (1-\tau)w_i \Rightarrow w_i = N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{\frac{-1}{\omega}} \quad \frac{PC^\gamma}{1-\tau}$$

• $\frac{\partial L}{\partial M}$:

$$\left(\frac{M}{P} \right)^{-\delta} - \lambda = 0 \Rightarrow \left(\frac{M}{P} \right)^{-\delta} = C^{-\gamma}$$

Firms:

From the consumption basket we know:

$$n_i = \left(\frac{CP^\rho}{A_i} \right) (p_i)^{-\rho}$$

so maximizing the profit $\pi_i = p_i y_i - w_i n_i$ result in:

$\partial \pi_i / \partial p_i$:

$$p_i A_i \quad \partial n_i / \partial p_i + A_i n_i - w_i \quad \partial n_i / \partial p_i = 0 \Rightarrow -\rho p_i \left(\frac{A_i n_i}{p_i} \right) + A_i n_i + \rho w_i \left(\frac{n_i}{p_i} \right) = 0$$

$$A_i (1-\rho) + \rho \frac{w_i}{p_i} = 0 \Rightarrow p_i = \frac{\rho}{\rho-1} \quad \frac{w_i}{A_i}$$

References

- Abel, A. B. (1983). Optimal investment under uncertainty. *The American Economic Review*, 73(1):228–233.
- Amador, M. and Weill, P.-O. (2010). Learning from prices: Public communication and welfare. *Journal of Political Economy*, 118(5):866–907.
- Angeletos, G.-M., Iovino, L., and La’O, J. (2020). Learning over the business cycle: Policy implications. *Journal of Economic Theory*, 190:105115.
- Angeletos, G.-M. and La’O, J. (2020). Optimal Monetary Policy with Informational Frictions. *Journal of Political Economy*, 128(3):1027–1064.
- Arellano, C., Bai, Y., and Kehoe, P. J. (2016). Financial frictions and fluctuations in volatility. (22990).
- Bachmann, R. and Bayer, C. (2013). ‘Wait-and-See’ business cycles? *Journal of Monetary Economics*, 60(6):704–719.
- Bachmann, R. and Bayer, C. (2014). Investment dispersion and the business cycle. *American Economic Review*, 104(4):1392–1416.
- Baker, S. R., Bloom, N., and Davis, S. J. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, 131(4):1593–1636.
- Basu, S. and Bundick, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, 85(3):937–958.
- Berger, D. and Vavra, J. (2017). Shocks vs. responsiveness: What drives time-varying dispersion? (23143).
- Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 77(3):623–685.
- Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, 86(3):1031–1065.
- Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk shocks. *American Economic Review*, 104(1):27–65.
- Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J. F., and Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6):2530–61.
- Gaballo, G. (2016). Rational inattention to news: The perils of forward guidance. *American Economic Journal: Macroeconomics*, 8(1):42–97.
- Hartman, R. (1972). The effects of price and cost uncertainty on investment. *Journal of Economic Theory*, 5(2):258–266.
- Jurado, K., Ludvigson, S. C., and Ng, S. (2015). Measuring uncertainty. *American Economic Review*, 105(3):1177–1216.
- Kehrig, M. (2015). The cyclical nature of the productivity distribution. *Working Paper*.

- Kohlhas, A. (2022). Learning by sharing: Monetary policy and common knowledge. *American Economic Journal: Macroeconomics*.
- La'O, J. and Tahbaz-Salehi, A. (2022). Optimal monetary policy in production networks. *Econometrica*, 90(3):1295–1336.
- Leduc, S. and Liu, Z. (2016). Uncertainty shocks are aggregate demand shocks. *Journal of Monetary Economics*, 82(C):20–35.
- Mackowiak, B. and Wiederholt, M. (2009). Optimal sticky prices under rational inattention. *American Economic Review*, 99(3):769–803.
- Mankiw, N. G. and Reis, R. (2002). Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, 117(4):1295–1328.
- Morris, S. and Shin, H. S. (2005). Central bank transparency and the signal value of prices. *Brookings Papers on Economic Activity*, 36(2):1–66.
- Oi, W. Y. (1961). The desirability of price instability under perfect competition. *Econometrica*, 29(1):58–64.
- Ou, S., Zhang, D., and Zhang, R. (2021). Information frictions, monetary policy, and the paradox of price flexibility. *Journal of Monetary Economics*, 120:70–82.
- Ravn, M. and Sterk, V. (2017). Job uncertainty and deep recessions. *Journal of Monetary Economics*, 90(C):125–141.